

Tenth Bielefeld-SNU joint Workshop in Mathematics

Feb 21 – Feb 24

Tuesday, February 21

Boundary value problems on non-Lipschitz uniform domains: Stability, compactness and the existence of optimal shapes

Michael Hinz (Bielefeld)

The talk deals with second order PDE in variational formulation on rough domains. We discuss boundary value problems for families of confined uniform (ε, ∞) -domains with (possibly) fractal boundaries and present novel results on convergence, compactness, spectral continuity and the existence of energy minimizing shapes. Our focus is on mixed and Robin problems, they involve integrals with respect to (possibly) fractal measures on the boundaries. These types of problems are relevant in linear and nonlinear acoustics, and we briefly mention related applications.

Title: Multi-index approach to singular SPDEs

Pavlos Tsatsoulis (Bielefeld)

In this talk I will give an introduction to the theory of singular Stochastic Partial Differential Equations (SPDEs) and discuss an alternative approach to Hairer's Regularity Structures by Otto–Sauer–Smith–Weber using multi-indices instead of trees. I will mainly focus on the analytical aspects of solution theory to singular SPDEs. Fragments of the talk will be based on the joint work “A diagram-free approach to the stochastic estimates in Regularity Structures” with F. Otto, P. Linares, and M. Tempelmayr.

Continuum limit of Lohe model on some class of Lie groups

Myeongju Kang (Seoul)

The Lohe model is a generalized high dimensional Kuramoto-type model whose oscillators lie on a special class of matrix Lie group. The continuum Lohe model is a time-evolutionary integro-differential equation which governs the Lohe phase field, and solution to the Lohe model becomes a simple function valued solution to the continuum Lohe model. In this talk, we study the emergent dynamics and global well-posedness of the continuum Lohe model which can be obtained by a continuum limit of the Lohe model. We first construct a local solution to the continuum Lohe model. Then, we find an invariant set to extend our local solution to a global one. Lastly, we show that sequence of simple functions obtained from the Lohe model converges to a solution of the continuum Lohe model in supreme norm sense. This talk is based on the joint work with Hangjun Cho and Seung-Yeal Ha.

Nonlinear Fokker-Planck equations with fractional Laplacian and McKean-Vlasov SDEs with Lévy-noise

Michael Röckner (Bielefeld)

This talk is concerned with the existence of mild solutions to nonlinear Fokker–Planck equations with fractional Laplace operator $(-\Delta)^s$ for $s \in (\frac{1}{2}, 1)$. The uniqueness of Schwartz distributional solutions is also proved under suitable assumptions on diffusion and drift terms. As applications, weak existence and uniqueness of solutions to McKean–Vlasov equations with Lévy–Noise, as well as the Markov property for their laws are proved.

Joint work with: Viorel Barbu, Al.I. Cuza University and Octav Mayer Institute of Mathematics of Romanian Academy, Iași, Romania

Numerical analysis on *regularization by noise* for singular SDEs

Chengcheng Ling (Vienna)

Recently numerous studies on *Regularization By Noise* demonstrate the positive effect from noise for the whole system. In the context of singular SDEs, initially it was introduced as under the regularization of the driven noise (e.g. a Brownian motion, an α -stable process, a fractional Brownian motion, etc), a random equation with singular (i.e. non-Lipschitz continuous) coefficients interestingly is well-posed, that is to say, there exists a unique solution to the equation and the solution satisfies some kind of stability estimates. Once the existence and uniqueness of the solution are shown theoretically, it is natural to ask how the solution looks like, whether it can be simulated. In this talk, we introduce two numerical methods in this spirit: Euler-Maruyama approximation and Milstein method (also called higher order scheme approximation). We derive optimal strong convergence rate for the Euler-Maruyama approximation without imposing any continuity assumption on the drift, under 'Krylov-Röckner' (or 'LPS') condition only. In the case of Milstein approximation, we show that the Hölder or Sobolev regularity of the drift can be leveraged to improve the convergence rate: drift with regularity of order $\alpha \in (0, 1)$ leads to convergence rate $\frac{1+\alpha}{2}$. Those results show us the *regularization by noise* effect from numerical viewpoint.

Measure-valued branching processes and nonlinear evolution equations on measures

Lucian Beznea (Bucharest)

On the set (denoted by M) of all finite measures on the d -dimensional Euclidean space we consider the evolution equation associated with the nonlinear equation $LF' + \sum_k b_k F^k = 0$, where L is the generator of a Markov process X on the d -dimensional Euclidean space and F' is the variational derivative of the function F defined on M . We show that it has a solution represented by means of the distribution of X and the nonlocal branching process on the finite configurations of M , induced by the probability $(b_k)_k$ on the natural numbers.

Random Voronoi-type tessellations

Anna Gusakova (Münster)

A tessellation in \mathbb{R}^d is a countable locally-finite collection of convex polytopes, which cover the space and have disjoint interiors. Random tessellations are among the most central objects studied in stochastic geometry. Their analysis is motivated by their rich inner-mathematical structures, but in equal measure also by the wide range of applications in which they arise. However, there are only very few mathematically tractable models for which rigorous results are available and which do not require an analysis purely by computer simulations.

In this talk we consider a few models of this kind, among which is a classical Poisson-Voronoi tessellation, whose construction is based on the homogeneous Poisson point process. The other three models, called β -, β' - and Gaussian-Voronoi tessellations, has been introduced recently and their construction is based on a space-time paraboloid hull process and generalizes that of the classical Poisson-Voronoi tessellation.

Based on the joint work with Christoph Thäle and Zakhar Kabluchko.

Wednesday, February 22

How singular are structured random matrices?

Torben Krüger (Erlangen)

The condition number of a high dimensional symmetric random matrix with independent and identically distributed entries above its diagonal (Wigner matrix) is typically of the order of the dimension N of this matrix. The reason for this relation is that zero lies in the bulk of the spectrum, where the spectral density is bounded and nonvanishing. Structured random matrices with blocks that allow for varying values of the entry variances from block to block and even zero blocks, may have divergent spectral densities at zero, leading to larger condition numbers. We expect the behaviour of these condition numbers to have a simple relation to the type of observed density blow-up. For block matrices with independent blocks, we classify all possible divergencies of the density, which all follow an inverse fractional power law whose exponent is $\frac{k}{(k+2)}$ for some positive integer k .

Fermions in a D-dimensional trap and universality in random matrix theory

Gernot Akemann (Bielefeld)

It has been shown that the ground state of N noninteracting Fermions in a harmonic trap can be described by random matrix theory. In $D = 1$ the harmonic oscillator is related to the Gaussian Unitary Ensemble (GUE). For $D = 2$ and a rotating trap, using complex coordinates the corresponding ensemble is non-Hermitian, the complex Ginibre ensemble. Its complex eigenvalues also enjoy an interpretation as a two-dimensional Coulomb gas, and many other applications. I will give a brief introduction into what can be computed using random matrices, including the mean and variance of the number of particles in a domain. The variance does not depend on the choice of a Gaussian potential and is thus universal for $N \rightarrow \infty$. This is joint work with Sungsoo Byun, Markus Ebke and Gregory Schehr.

Hydrodynamic limits of the relativistic quantum equations with gauge fields

Jeongho Kim (Seoul)

In this talk, we discuss the hydrodynamic limit (or semi-classical limit) of the Maxwell-Klein-Gordon equations. In the first part of the talk, we introduce the concept of hydrodynamic limit of the Schrodinger-type equations and explain the main idea of the modulated energy method. Then, we focus on the Maxwell-Klein-Gordon equations, which describe the dynamics of the matter field under the electromagnetic fields in 3D. We introduce an appropriate modulated energy for the Maxwell-Klein-Gordon equations, and provide the hydrodynamic limit estimate of it.

Limit theorems for time averages of continuous-state branching processes with immigration

Peter Kuchling (Wuppertal)

In this talk, we investigate limit theorems for the time-averaged process of a subcritical continuous-state branching process with immigration starting in some nonnegative state. Under a second moment condition on the branching and immigration measures, we obtain the classical limit theorems. Assuming additionally that the big jumps of the branching and immigration measures have finite exponential moments of some order, we prove in our main result the large deviation principle and provide a semi-explicit expression for the good rate function in terms of the branching and immigration mechanisms. Our methods are deeply based on a detailed study of the corresponding generalized Riccati equation and related exponential moments of the time-averaged process. Joint work with Mariem Abdellatif, Martin Friesen and Barbara Rüdiger.

Thursday, February 23

Homogenizations :Competing of scales

Ki-Ahm Lee (Seoul)

In this talk, we are going to consider the competing two different scales in the theory of Homogenizations. For example, competitions between scale in space and time variables arise in the homogenization of parabolic equations, where different averaging processes happen based on the different order of scales. Similar phenomenon happens in highly oscillating obstacles where the periodicity and the thickness of obstacles compete each other. We are going to discuss regularities of the solution and the effective equations satisfied by the limits.

The Wiener criterion for fully nonlinear elliptic equations

Sechan Lee (Seoul)

In this talk, we discuss about the boundary continuity of solutions to fully nonlinear elliptic equations. We first introduce a capacity for operators in non-divergence form and derive several capacity estimates. We next develop the Wiener criterion, which characterizes a regular boundary point via potential theory. Our approach utilizes the asymptotic behavior of homogeneous solutions, together with Harnack inequality and the comparison principle.

Smoothing properties of averages over curves

Sanghyuk Lee (Seoul)

In this talk, we discuss recent progress in the study of smoothing properties of the averaging operator defined by convolution with a measure on a smooth curve.

Maxwell equations on domains with perfectly conducting boundary conditions

Robert Schippa (Karlsruhe)

We consider Maxwell equations on a smooth domain with perfectly conducting boundary conditions in isotropic media. In the charge-free case we recover Strichartz estimates for wave equations on domains due to Blair–Smith–Sogge for Maxwell equations on domains. In two dimensions, the local well-posedness of the Maxwell system with Kerr nonlinearity is improved via Strichartz estimates.

This is joint work with Nicolas Burq (Université Paris-Saclay).

Heat kernel estimates for Dirichlet forms vanishing at the boundary

Soobin Cho (Seoul)

In this talk, I will discuss heat kernel estimates for jump-type Dirichlet forms and corresponding Markov jump processes with jump kernels vanishing at the boundary. I will give an overview of results on heat kernel estimates for jump processes in spaces with boundaries such as actively reflected processes, killed processes and censored processes. Then I will present some new features of jump processes whose jump kernel vanishes at the boundary. The talk is based on joint work with Panki Kim, Renming Song and Zoran Vondracek.

Robust near-diagonal Green function estimates

Minhyun Kim (Bielefeld)

In this talk, we study sharp near-diagonal pointwise bounds for the Green function for nonlocal operators of fractional order $\alpha \in (0, 2)$. The novelty of our results is two-fold: the estimates are robust as $\alpha \rightarrow 2-$ and we prove the bounds without making use of the Dirichlet heat kernel. In this way we can cover cases, in which the Green function satisfies isotropic bounds but the heat kernel does not.

Nonlocal Sobolev spaces and application to integrodifferential equations

Guy Fabrice Foghem Gounoue (Dresden)

Within the framework of Hilbert spaces, we introduce and study certain nonlocal Sobolev spaces on domains, tailor made for nonlocal IntegroDifferential Equations (IDEs) driven Levy type operators. As application, we study IDEs with prescribed Dirichlet, Neumann or Robin conditions on the complement of a domain. A prototypical example of a nonlocal Levy operator includes the well known fractional Laplace operator. We also prove that local weak solutions of the classical PDEs with Dirichlet, Neumann conditions are L^2 limits of nonlocal weak solutions of the corresponding IDEs.

Friday, February 24

Soliton resolution for equivariant self-dual Chern-Simon-Schrödinger equation

Soonsik Kwon (Daejeon)

I will present joint works with Kihyun Kim and Sung-Jin Oh on the global behaviors of self-dual Chern-Simon-Schrödinger equations (CSS) under equivariance symmetry, with emphasis on the soliton resolution. It is known that (CSS) admits solitons and finite-time blow-up solutions. In this work, we show that any solution with equivariant data in weighted Sobolev space $H^{1,1}$ decomposes into at most one modulated soliton and a radiation part. A striking fact is that the nonscattering part must be a single modulated soliton. This is mainly due to the defocusing nature of the equation in the exterior of a soliton profile.

Rigidity of smooth finite-time blow-up for the self-dual Chern-Simons-Schrödinger equation within equivariance

Kihyun Kim (Paris)

We continue to consider the long time dynamics for the self-dual Chern-Simons-Schrödinger equation (CSS) within equivariant symmetry as a continuation of the previous talk, where we saw that any finite energy finite-time blow-up solutions to (CSS) decompose into precisely one modulated soliton and a radiation. In this talk, we address an even stronger rigidity result in the high equivariance case (i.e., the equivariance index ≥ 1): any smooth finite-time blow-up solutions have a universal blow-up speed, i.e., the pseudoconformal one. We explore this rigid dynamics using modulation analysis.

Scattering of cubic Dirac equations with a general class of Hartree-type nonlinearity for the critical Sobolev data

Seokchang Hong (Seoul)

Recently low-regularity behaviour of solutions to cubic Dirac equations with the Hartree-type nonlinearity has been extensively studied in somewhat a specific assumption on the structure of the nonlinearity. The key approach of previous results was to exploit the null structure in the nonlinearity and the decay of the Yukawa potential. In this talk, we aim to go beyond; we investigate the strong scattering property of cubic Dirac equations with quite a general class of the Hartree-type nonlinearity, which covers the Coulomb potential as well as the Yukawa potential, and the bilinear form, in which one cannot use the specific null structure. As a direct application, we also obtain the scattering for the boson-star equations with the scaling-critical Sobolev data.

The heat flow of random polynomials and the GAF

Jonas Jalowy (Münster)

Start with a random polynomial with i.i.d. (rescaled) coefficients and look at the empirical distribution of their (complex) roots. In this talk, we will investigate the evolution of the roots when the polynomial (or the Gaussian analytic function) undergoes the heat flow. In one prominent example of Weyl polynomials, the limiting distribution evolves from the circular law into the elliptic law until it collapses to the semicircular distribution. We will describe the limiting distribution and discuss the dynamics of the roots from various remarkable perspectives, accompanied by illustrative simulations. The talk is based on a joint work in progress with Brian Hall, Ching Wei Ho and Zakhar Kabluchko.

Gradient estimates for mixed local and nonlocal problems

Ho-Sik Lee (Seoul)

Mixed local and nonlocal problems are kinds of intensively studied topics in the realm of the regularity theory of PDE recently.

In this talk, we present Calderon-Zygmund type estimates for mixed local and nonlocal problems.

Assuming the minimal conditions on boundary and nonlinearity, we use the appropriate comparison estimates motivated on the paper [C. De Filippis and G. Mingione, *Math. Ann.*, to appear].

Potential theory of Dirichlet forms with jump kernels blowing up at the boundary

Panki Kim (Seoul)

In this talk we study the potential theory of Dirichlet forms on the half-space defined by the jump kernel $J(x, y) = |x - y|^{-d-\alpha} B(x, y)$ and the killing potential $\kappa x_d^{-\alpha}$, where $\alpha \in (0, 2)$ and $B(x, y)$ can blow up to infinity at the boundary. The jump kernel and the killing potential depend on several parameters. For all admissible values of the parameters involved, the boundary Harnack principle holds. This is a joint work with Renming Song and Zoran Vondraček.