Eighth Bielefeld-SNU joint Workshop in Mathematics

Feb 24 – Feb 26

Monday, February 24

Global gradient estimates for nonuniformly elliptic equations

Sun-Sig Byun

A very general class of nonuniformly elliptic equations with discontinuous nonlinearities over nonsmooth domains is considered for the study of global gradient estimates in the setting of Musielak-Orlicz spaces.

Regularization by noise for SDEs and related systems: a tale of two approaches

Oleg Butkovsky

(based on the joint works of the author with S. Athreya, K. Dareiotis, M. Gerencsér, K. Le, L. Mytnik)

I will talk about two complementing approaches to studying regularization by noise for stochastic systems. The first one is based on a PDE technique (the so-called Zvonkin-Veretennikov transformation) and dates back to 1980s. We will see how a new version of this method can be used for studying SDEs with distributional drift. This approach is usually quite robust, however it is not applicable if Ito's formula is not available (e.g., for SPDEs or SDEs driven by fractional BM). The second approach is more flexible and is based on the newly developed stochastic sewing lemma (SSL) of Le (2018). We develop a new version of SSL, which is suitable for SPDEs, and show existence and uniqueness of solutions to the stochastic heat equation with the distributional drift. We will also provide applications to numerical methods and show convergence of the Euler scheme for SDEs driven by fractional BM with non-regular
drift.

[1] S. Athreya, O. Butkovsky, L. Mytnik (2018). Strong existence and uniqueness for stable stochastic differential equations with distributional drift. arXiv:1801.03473, to appear in the Annals of Probability.
[2] O. Butkovsky, K. Dareiotis, M. Gerencsér (2019). Approximation of SDEs – a stochastic sewing approach. arXiv:1909.07961.

A stochastic Gronwall lemma and well-posedness of stochastic delay equations

Michael Scheutzow

We present a stochastic Gronwall lemma associated to a possibly discontinuous local martingale which generalizes earlier versions of the inequality in the continuous case. Then we show how the lemma can be used to prove existence and uniqueness of solutions of a stochastic delay equation driven by a cadlag local martingale.
This is joint work with Sima Mehri (Berlin).

Strichartz estimates for orthonormal families of initial data

Sanghyuk Lee

This talk concerns the Strichartz estimates for orthonormal families of initial data which generalize the classical Strichartz estimates. These estimates have their motivation in understanding a system of infinitely many equations describing infinitely many particles. We prove new estimates in the case of the wave, Klein–Gordon and fractional Schrödinger equations. Due to a certain technical barrier, except for the classical Schrödinger equation, the Strichartz estimates for orthonormal families of initial data have not previously been established up to the sharp summability exponents in the full range of admissible pairs. We obtain the optimal estimates in various notable cases and improve the previous results. The main novelty is the use of estimates for weighted oscillatory integral which we believe to be of wider independent interest. This talk is based on joint work with Neal Bez and Shohei Nakamura.

Estimates on transition densities of subordinators with jumping density decaying in mixed polynomial orders

Panki Kim

In this talk, we discuss the sharp two-sided estimates on the transition densities for subordinators whose Lévy measures are absolutely continuous and decaying in mixed polynomial orders. Under a weaker assumption on Lévy measures, we also discuss a precise asymptotic behaviors of the transition densities at infinity. Our results cover geometric stable subordinators, Gamma subordinators and much more. This is a joint work with Soobin Cho.

Law of iterated logarithms for symmetric Dirichlet form

Jaehun Lee

In this talk, we discuss the law of iterated logarithm for symmetric Hunt processes in metric measure space equipped with volume doubling and reverse volume doubling conditions. For the Hunt process $X = (X_t)_{t \geq 0}$, we focus on the law of iterated logarithms for the distance $d(X_0, X_s)$ and local time $\ell(x, t)$. Especially, we concentrate on the sharp sufficient conditions when time goes to 0 or ∞.

This is joint work with Soobin Cho and Panki Kim.

Heat kernel estimates for mixed systems of diffusions and jump processes

Jaehoon Kang

Let $Z = (Z^1, Z^2, \ldots, Z^{d+n})$ be a symmetric Markov process on $\mathbb{R}^{d+n}$, where $Z^1, Z^2, \ldots, Z^d$ are symmetric one-dimensional α-stable processes and $Z^{d+1}, \ldots, Z^{d+n}$ are one-dimensional Brownian motions. We assume all components to be independent.

Let $(\mathcal{E}^Z, \mathcal{F}^Z)$ be the Dirichlet form associated with Z. We study Dirichlet forms $(\mathcal{E}, \mathcal{F})$ that are derived from $(\mathcal{E}^Z, \mathcal{F}^Z)$ by allowing for bounded measurable coefficients. We discuss heat kernel estimates for the symmetric Markov process X that is associated with $(\mathcal{E}, \mathcal{F})$. In particular, we show that the heat kernel is comparable to the one of $(\mathcal{E}^Z, \mathcal{F}^Z)$. The result can be seen as a robustness result for heat kernels like the one of Aronson (1968) for diffusions and the one of Chen/Kumagai (2003) for symmetric jump processes.

The talk is based on a joint work with Moritz Kassmann.

Tuesday, February 25

On boundary confinements for the Coulomb gas

Nam-Gyu Kang

In the theory of random Hermitian matrices, it is well known that the Airy kernel and the Bessel kernel describe spacing at the soft and the hard edge of the spectrum, respectively. In this talk, I will introduce a family of boundary confinements for Coulomb gas ensembles, which interpolates between the free boundary and hard edge cases. I will discuss the edge universality for these ensembles when the underlying potential is radially symmetric. This is based on joint work with Yacin Ameur and Seong-Mi Seo.

Construction of skew-orthogonal polynomials from orthogonal polynomials for quaternionic non-hermitian random matrices

Markus Ebke

Skew-orthogonal polynomials can be used to determine the eigenvalue correlation functions of quaternionic non-hermitian random matrices. But finding the right set of polynomials for a given matrix ensemble is difficult. In my talk I will present a construction starting from the orthogonal polynomials of the complex non-hermitian ensemble, which is much better understood. I will show that if these orthogonal polynomials fulfill a three-step recurrence relation, then finding the sought after skew-orthogonal polynomials is easy.

Stochastic analysis of ensemble-based Kalman-type filters

Wilhelm Stannat

Ensemble-based Kalman-type filters form a large class of stochastic algorithms for the Bayesian state estimation of Markovian signals observed with noise. They have been invented in the context of numerical weather prediction and since then successfully applied to various estimation problems for time-structured data in the engineering and natural sciences. The theoretical understanding of many of their features, however, are still in their infancy.

In our talk we will present asymptotic results for Ensemble-based Kalman-type filters in the continuous time limit, thereby providing a rigorous mathematical derivation of the Ensemble Kalman-Bucy filter including approximation errors. A striking implication is the universality of the time-continuous limit across a large class of Ensemble-based Kalman-type filters.

We further investigate the behavior of the resulting Ensemble Kalman-Bucy filter applied to continuous-time filtering problems. We also derive mean field limiting equations in the infinite ensemble size limit to assess asymptotic consistency, as well as uniform-in-time accuracy and stability results for finite ensemble sizes.

References

[1] Long-time stability and accuracy of the ensemble Kalman-Bucy filter for fully observed processes and small measurement noise (with J. de Wiljes, S. Reich), SIAM J. Appl. Dyn. Syst., Vol. 17, 1152-1181, 2018.

[2] On the continuous time limit of the Ensemble Kalman Filter (with T. Lange), Preprint arXiv:1901.05204, submitted.

[3] On the continuous time limit of Ensemble Square Root Filters (with T. Lange), Preprint arXiv:1910.12493, submitted.

Taming the MHD equations

Andre Schenke

In this talk, a new model for the flow of an electrically conducting fluid through a porous medium is presented: the *tamed magnetohydrodynamics (TMHD) equations*. After a brief discussion of regularisation schemes and motivation for fluid dynamical equations, we prove existence, uniqueness and regularity in the deterministic case. When the taming parameter tends to infinity, the solution to the TMHD equations converges in a suitable sense to a weak solution of the MHD equations. In the stochastic case, we prove existence and uniqueness of a solution for periodic boundary conditions, as well as existence of a Feller semigroup and an invariant measure. Finally, we discuss questions of uniqueness and ergodicity for this invariant measure.

A review on time-homogeneous Itô-SDEs with locally integrable coefficients

Gerald Trutnau

We present a review on our recent results on time-homogeneous Itô-SDEs with locally integrable coefficients. We shall consider weak existence, uniqueness (pathwise and in law), Krylov type estimates, criteria for non-explosion, recurrence, existence, non-existence, and uniqueness of finite and σ-finite invariant measures. This is based on joint work with Haesung Lee (Bielefeld University).

Well-posedness for a class of degenerate Itô-SDEs with fully discontinuous coefficients

Haesung Lee

We show uniqueness in law for a general class of stochastic differential equations in $\mathbb{R}^d$, $d \geq 2$, with possibly degenerate and/or fully discontinuous locally bounded coefficients among all weak solutions that spend zero time at the points of degeneracy of the dispersion matrix. The points of degeneracy have d-dimensional Lebesgue-Borel measure zero. Weak existence is obtained for more general, not necessarily locally bounded drift coefficient. Generalized Dirichlet form theory and regularity results for PDEs are the main tools to derive our results. This is joint work with Gerald Trutnau (Seoul National University).

On potential theory of Markov processes with jump kernels decaying at the boundary

Zoran Vondraček

In this talk, I will consider some potential theory of the process Y on an open set $D \subset \mathbb{R}^d$ associated with a pure jump Dirichlet form whose jump kernel has the form $J(x,y) = B(x,y)|x-y|^{-d-\alpha}$, $0 < \alpha < 2$. Here $B(x,y)$ – the boundary term – depends on $\delta_D(x), \delta_D(y)$ and $|x-y|$, and is allowed to approach 0 at the boundary. This is in contrast with previous works where $B(x,y)$ is assumed to be bounded between two positive constants, which can be viewed as a uniform ellipticity condition for non-local operators. The conditions and the form of the boundary term $B(x,y)$ are motivated by jump kernels of some subordinate killed Lévy processes.

We prove that non-negative harmonic functions of the process satisfy the Harnack inequality and Carleson's estimate. Furthermore, in case when D is the half-space we investigate when the boundary Harnack principle holds. This is joint work with Panki Kim (Seoul National University) and Renming Song (University of Illinois).

Wednesday, February 26

Boundary regularity of local and nonlocal equations

Ki-Ahm Lee

In this talk, we are going to discuss boundary regularities of various degenerate local equation and nonlocal equations. Diffusion rates deform undefined geometry related to diffusion and the corresponding distance function makes important role in the theory of regularity. And then we will also discuss the possible applications.

Hölder Estimates for Parabolic Nonlocal Operators

Marvin Weidner

In this talk we study parabolic equations determined by nonlocal operators in a general framework of bounded and measurable coefficients. Our emphasis is on the weak Harnack inequality and Hölder regularity estimates for solutions of such equations. We allow the underlying jump measures to be singular with a singularity that depends on the coordinate direction. This approach also allows to study several classes of non-singular jump measures that have not been dealt with so far. This is joint work with Moritz Kassmann (Bielefeld University) and Jamil Chaker (University of Chicago).

Heat kernel bounds for non-local operators with singular kernels

Kyung-Youn Kim

We prove sharp two-sided bounds of the fundamental solution for an integro-differential operator of order $\alpha \in (0, 2)$ that generates a d-dimensional Markov process. This process can be written as d independent copies of one-dimensional jump processes, i.e., the jumping measure is singular with respect to the d-dimensional Lebesgue measure. This is a joint work with Moritz Kassmann and Takashi Kumagai.