Analytic theory of Itô-stochastic differential equations with non-smooth coefficients

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I will present a review, from a mainly analytic point of view, on time-homogeneous Itô-stochastic differential equations with locally integrable coefficients ([1]).
Analytic tools will be provided to describe local and global behavior of solutions to Itô-stochastic differential equations with non-degenerate Sobolev diffusion coefficients and locally integrable drift. Regularity theory of partial differential equations is applied to construct such solutions and to obtain strong Feller properties, irreducibility, Krylov-type estimates, moment inequalities, various types of non-explosion criteria, and long time behavior, e.g., transience, recurrence, and convergence to stationarity. The approach is based on the realization of the transition semigroup associated with the solution of a stochastic differential equation as a strongly continuous semigroup in the L^p-space with respect to a weight that plays the role of a sub-stationary or stationary density. This way we obtain in particular a rigorous functional analytic description of the generator of the solution of a stochastic differential equation and its full domain. The existence of such a weight is shown under broad assumptions on the coefficients. A remarkable fact is that although the weight may not be unique, many important results are independent of it. Given such a weight and semigroup, one can construct and further analyze in detail a weak solution to the stochastic differential equation combining variational techniques, regularity theory for partial differential equations, potential, and generalized Dirichlet form theory. Under classical-like or various other criteria for non-explosion we obtain the existence of a pathwise unique and strong solution with an infinite lifetime. These results substantially supplement the classical case of locally Lipschitz or monotone coefficients.
In principle this course requires knowledge from several different areas of mathematics (probability theory, stochastic differential equations, PDE, functional analysis, semigroup and operator theory, (generalized) Dirichlet form theory). Therefore, I will try to focus more on connections and heuristics, rather than on technical aspects and will aim at adapting to the preferences of the participants.
An update with a more detailed plan of the course will be given shortly before its start. If you have questions, please email me at any time.

References

[1] H. Lee, W. Stannat, G. Trutnau, *Analytic theory of Itô-stochastic differential equations with non-smooth coefficients*, SpringerBriefs in Probability and Mathematical Statistics. Springer, Singapore, 2022.

Additional references will be mentioned during the course.