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Abstracts

Miroslav Bulíček
(Charles University, Prague)

On nonlinear elliptic equations beyond the natural duality pairing

We establish existence, uniqueness and optimal regularity results for very weak solutions to certain nonlinear elliptic boundary value problems. We provide a unified approach that leads qualitatively to the same theory as that one available for linear elliptic problems with continuous coefficients, e.g. the Poisson equation. The result is based on several novel tools that are of independent interest: local and global estimates for (non)linear elliptic systems in weighted Lebesgue spaces with Muckenhoupt weights, a generalization of the celebrated divcurl lemma for identification of a weak limit in border line spaces and the introduction of a Lipschitz approximation that is stable in weighted Sobolev spaces. Furthemore, we generalize the technique in order to cover also the nonlinear Stokes-like problems.

Sun-Sig Byun
(Seoul National University)

Recent regularity results of solutions to nonlinear elliptic and parabolic equations with nonstandard growth

We discuss recent advances in the regularity theory of solutions to nonlinear elliptic and parabolic equations with nonstandard growth.

Lars Diening
(University of Osnabrück)

Linearization of the p-Poisson equation

This is a joint work with Massimo Fornasier and Maximilian Wank. In this talk we propose a iterative method to solve the non-linear p-Poisson equation. The method is derived from a relaxed energy by an alternating direction method. We are able to show algebraic convergence of the iterates to the solution. However, our numerical experiments based on finite elements indicate optimal, exponential convergence.

Dominik Dier
(Ulm University)

Discrete versions of the Li-Yau gradient estimate

We study positive solutions to the heat equation on graphs. We prove variants of the Li-Yau gradient estimate and the differential Harnack inequality. For some graphs, we can show the estimates to be sharp. We establish new computation rules for differential operators on discrete spaces and introduce a relaxation function that governs the time dependency in the differential Harnack estimate.
Joint work with Moritz Kassmann and Rico Zacher.

Hongjie Dong
(Brown University)

Schauder and Dini type estimates for nonlocal fully nonlinear equations

In 1934, J. Schauder first established by now well-known Schauder estimates for linear elliptic equations, which became an indispensable tool in the theory of partial differential equations. For fully nonlinear concave elliptic equations, such result was obtained by M. V. Safonov in 1988, following the seminal work of L. C. Evans and N. V. Krylov in early 1980s. I will present some recent work on Schauder estimates for a class of concave fully nonlinear nonlocal elliptic and parabolic equations with rough and non-symmetric kernels, where the data are allowed to be bounded and measurable. A further Dini type estimate will also be discussed.

Torben Fattler
(University of Kaiserslautern)

Analysis of the stochastic quantization for a fractional polymer measure

We prove existence of a diffusion process whose invariant measure is the fractional polymer or Edwards measure for fractional Brownian motion in dimension $d \in \mathbb{N}$ with Hurst parameter $H \in (0,1)$ fulfilling $dH < 1$. The diffusion is constructed via Dirichlet form techniques in infinite dimensional (Gaussian) analysis. By providing a Fukushima decomposition for the stochastic quantization of the fractional Edwards measure we show that the constructed process solves weakly a stochastic differential equation in infinite dimension for quasiall starting points. Moreover, the solution process is driven by an OrnsteinUhlenbeck process taking values in an infinite dimensional space and is unique, in the sense that the underlying Dirichlet form is Markov unique. The equilibrium measure, which is by construction the fractional Edwards measure, is specified to be an extremal Gibbs state. The talk bases on a jointed work with W. Bock and L. Streit.

Jens Frehse
(University of Bonn)

Bellman equations with mean-field dependence

Bellman equations to stochastic optimal control problems and corresponding games can be derived via a purely analytical, stochastic-free way by minimizing the so called Vlasov-McKean-functionals.

These functionals are defined by space-time integrals with integrand $m(.;v)f(.,v) + a$ boundary term. Here v are the controll variables, f corresponds to the pay off, and m is the so called mean-field variable. The function m satisfies the heat equation perturbed by a drift term corresponding to the dynamic of the ODE system.

In this setting, the necessary condition for a minimizer leads to a Bellman equation without m dependence.

If one assumes an additional m dependence in the data, say $f = f(.,v,m)$, or an additional summand $sigma(m)$ augmenting the Vlasov-McKean-functional, the Bellman equation will depend on

m. Recently, some work has done in this field due to certain applications, but the analytical treatment is more complicated than in the classical case. Among others, the usual maximum principle is not available to obtain a starting estimate.

In this contribution, some of the analytical problems are discussed. As application, we present a stochastic control problem where an additional risk control is implemented.

Common papers with Alain Bensoussan and Dominic Breit.

Martin Grothaus
(University of Kaiserslautern)

Weak Poincaré inequalities for convergence rate of degenerate diffusion processes

For a contraction C_0-semigroup on a separable Hilbert space, the decay rate is estimated by using the weak Poincaré inequalities for the symmetric and anti-symmetric part of the generator. As applications, non-exponential convergence rate is characterized for a class of degenerate diffusion processes, so that the study of hypocoercivity is extended. Concrete examples are presented.

Michael Hinz
(Bielefeld University)

Essential self-adjointness of Laplacians and two-parameter processes

It is well known that if one removes a closed set from a complete Riemannian manifold, the Markov uniqueness of the Laplacian with smooth functions, compactly supported in the complement of that set, can be characterized using 1-capacities and hitting probabilities of Brownian motion. We study analogous characterizations for essential self-adjointness, then in terms of 2-capacities and, in cases we can treat, two-parameter processes. We also discuss related results for fractional Laplacians. The results are based on joint work with Jun Masamune.

Myungjoo Kang
(Seoul National University)

Image processing, Deep Learning and their applications

Since 1990, many mathematicians have been studied the image processing based on partial differential equations and variational methods. Using TV(total variation) and some optimization techniques, there were a lot of improvement in image processing areas. Until the deep learning coming out, those were the state of art methods in these areas. But once using the deep learning techniques, it turns out that, in almost every areas in image processing fields, deep learning is the best method. I will compare the results between the variational image processing techniques and deep learning. Also, I will talk about the industrial applications.

Moritz Kassmann
(Bielefeld University)

Nonlocal quadratic forms and kinetic equations

We report on recent developments that link kinetic equations to fractional Sobolev spaces. It turns out that comparability of corresponding quadratic forms is both, important and challenging. We present recent results based on joint works with B. Dyda and with K.-U. Bux and T. Schulze.

Matthias Keller
(University of Potsdam)

Optimal Hardy inequalities on graphs

We construct optimal Hardy weights on graphs by using positive superharmonic functions of the operator. This is joint work with Felix Pogorzelski and Yehuda Pinchover.

Kyun-youn Kim
(Bielefeld University)

Heat kernel estimates for symmetric Markov processes in $C^{1,\eta}$ open sets

We consider a large class of symmetric pure jump Markov processes dominated by isotropic unimodal Lévy processes with weak scaling conditions. We first establish sharp two-sided heat kernel estimates for these processes in $C^{1,\rho}$ open sets, $\rho \in (\overline{\alpha}/2, 1]$ where $\overline{\alpha}$ is the upper scaling parameter in the weak scaling conditions.

Ki-Ahm Lee
(Seoul National University)

Global solution in Curvature Flows

In this talk, we are going to consider the global existence of smooth graph in Gauss Curvatures where the local control of normal derivative is crucial step to show the preservation of the graph structure and then further regularity. And we will also discuss about the Hamilton's Conjecture on the existence of cigar type soliton with flat spots.

Erika Maringová
(Charles University, Prague)

Globally Lipschitz minimizers for variational problems with linear growth

The classical example of a variational problem with linear growth is the minimal surface problem. It is well known that for smooth data such problem posses a regular (up to the boundary) solution if the domain is convex (or has positive mean curvature). On the other hand, for non-convex domains we know that there always exist data for which the solution does exist only in the space BV (the desired trace is not attained). In the work we sharply identify the class of functionals (such that the minimal surface problem is equivalently described by a particular functional from this class) for which we always have regular (up to the boundary) solution in any dimension for arbitrary $C^{1,1}$ domain. Furthermore, we show that the class is sharp, i.e., whenever the functional does not belong to the class then we can find data for which the solution does not exist.
Joint work with Lisa Beck and Miroslav Bulíček.

Florentin Münch
(University of Potsdam)

Rigidity properties of the hypercube via Bakry Emery curvature

We give rigidity results for discrete Bonnet-Myers diameter bound and Lichnerowicz eigenvalue estimate. Both inequalities are sharp if and only if the underlying graph is a hypercube. The proofs use well-known semigoup methods as well as new direct methods which translate curvature to combinatorial properties. The results can be seen as first known discrete analogues of Cheng's and Obata's rigidity theorems.

Michael Röckner
(Bielefeld University)

Backward uniqueness for a class of SPDE

We present recent results on backward uniqueness of solutions to stochastic semilinear parabolic equations and also for the tamed 3D NavierStokes equations driven by linear multiplicative Gaussian noises. In the first case we use a rescaling transformation to reduce the SPDE to a random PDE. Applications to approximate controllability of nonlinear stochastic parabolic equations with initial controllers are given. The method of proof relies on the logarithmic convexity property known to hold for solutions to linear evolution equations in Hilbert spaces with self-adjoint principal part.
Joint work with Viorel Barbu.

Armin Schikorra
(University of Freiburg)

Free boundary problems for conformally invariant variational functions

I will present a regularity result at the free boundary for critical points of a large class of conformally invariant variational functionals. The main argument is that the Euler-Lagrange equation can be interpreted as a coupled system, one of local nature and one of nonlocal nature, and that both systems (and their coupling) exhibit an antisymmetric structure which leads to regularity estimates.

Gerald Trutnau
(Seoul National University)

Conservativeness criteria for generalized Dirichlet forms

We develop sufficient analytic conditions for conservativeness of non-sectorial perturbations of symmetric Dirichlet forms which can be represented through a carré du champ on a locally compact separable metric space. These form an important subclass of generalized Dirichlet forms. In case there exists an associated strong Feller process, the analytic conditions imply conservativeness, i.e. non-explosion of the associated process in the classical probabilistic sense. As an application of our general results on locally compact separable metric state spaces, we consider a generalized Dirichlet form given on a closed or open subset of $\mathbb{R}^d$ which is given as a divergence free first order perturbation of a symmetric energy form. Then using volume growth conditions of the carré du champ and the non-sectorial first order part, we derive an explicit criterion for conservativeness. We present several concrete examples which relate our results to previous ones obtained by different authors. In particular, we show that conservativeness can hold for a cubic variance if the drift is strong enough to compensate it. This is joint work with Minjung Gim (NIMS, South Korea) and continues our previous work on transience and recurrence of generalized Dirichlet forms.

Lauri Viitasaari
(Aalto University)

Generalised Skorokhod integrals and indefinite Wiener integrals with applications to backward stochastic differential equations

In this talk we introduce a generalised Skorokhod integrals with respect to a Gaussian process X. In particular, our generalised approach to Skorokhod integral covers, besides the classical Skorokhod integral, the extended divergence operator of [3] and the Wick-Itô integral of [1]. In addition, we discuss the concept of an indefinite Wiener integral. To be precise on the definition, we say that a given Gaussian process $X = (X_t)_{t\in[0,T]}$ has an indefinite Wiener integral if for every $r \in [0, T]$ there is a continuous linear operator $\mathbb{I}_r : \mathcal{H} \mapsto \mathcal{H}$ satisfying $\mathbb{I}_r \mathbf{1}_{(0,t]} = \mathbf{1}_{(0,r\wedge t]}$, where $\mathcal{H}$ is the Hilbert space spanned by the indicator functions $\mathbf{1}_{(0,t]}$ and closed with respect to the inner product $\langle \mathbf{1}_{(0,t]}, \mathbf{1}_{(0,s]} \rangle_{\mathcal{H}} = R(t,s)$, where R is the covariance function of X.

In this talk we discuss this property and give its geometrical meaning. We introduce several processes which has this property, and also explain how this can be checked for a given process. In particular, any linear combination of independent fractional Brownian motions with arbitrary Hurst parameters $H \in (0,1)$ have this property. Finally, we show the importance of the generalised Skorokhod integrals and indefinite Wiener integrals by revealing how these concepts can be applied to study backward stochastic differential equations driven by the corresponding Gaussian process. In the case of a fractional Brownian motion, this gives new understanding on backward stochastic differential equations driven by a fractional Brownian motion as well as the process itself. Comparison to a more classical approach and relation to partial differential equations will be discussed.

[1] Bender, C., *Backward SDEs driven by Gaussian processes*. Stochastic Process. Appl., 124: 2892–2916, 2014.

[2] Bender, C., Viitasaari, L., *A general non-existence result for linear BSDEs driven by Gaussian processes*. Stochastic Process. Appl., DOI: 10.1016/j.spa.2016.07.012, 2016.

[3] León, J.A., Nualart, D., *An extension of the divergence operator for Gaussian processes*. Stochastic Process. Appl., 115(3): 481–492, 2005.

Zoran Vondraček

(University of Zagreb)

Potential theory of subordinate killed Brownian motion

Let W^D be a killed Brownian motion in a domain $D \subset \mathbb{R}^d$ and S an independent subordinator with Laplace exponent ϕ. The process Y^D defined by $Y_t^D = W_{S_t}^D$ is called a subordinate killed Brownian motion. It is a Hunt process with infinitesimal generator $\phi(-\Delta|_D)$, where $\Delta|_D$ is the Dirichlet Laplacian. In this talk I will present several potential-theoretic results for Y^D under a weak scaling condition on the derivative of ϕ. These results include the scale invariant Harnack inequality for non-negative harmonic functions of Y^D, and two types of scale invariant boundary Harnack principles with explicit decay rates. The first boundary Harnack principle deals with a $C^{1,1}$ domain D and non-negative functions which are harmonic near the boundary of D, while the second one is for a more general domain D and non-negative functions which are harmonic near the boundary of an interior open subset of D. The obtained decay rates are not the same, reflecting different boundary and interior behaviors of Y^D. The results are new even for the case of the stable subordinator. (Joint work with P. Kim and R. Song)

Emil Wiedemann

(University of Hannover)

Weak-Strong Uniqueness in Fluid Dynamics

Various concepts of weak solution have been suggested for the fundamental equations of fluid dynamics over the last few decades. However, such weak solutions may be non-unique, or at least their uniqueness is unknown. Nevertheless, a conditional notion of uniqueness, the so-called weak-strong uniqueness, can be established in various situations. We present some recent results, both positive and negative, on weak-strong uniqueness in the realm of incompressible and compressible fluid dynamics. Applications to the convergence of numerical schemes will be indicated.