Probability : Sheet 2 (solution)

Problem 1

We have 80 people that watched the clip. Among these 80 people, 30% are interested by the product. Among the 120 people that didn't see the clip, $\frac{24}{120}$ are interested by the product. So, at the end,

$$\frac{0.3}{24/120} \approx 1.5,$$

what mean that

$$\frac{\text{Proportion of people that saw and the clip, like the product}}{\text{Proportion of people that didn't see the clip, like the product}} \approx 1.5,$$

In other words, seing the clip increase your interest on the product of 50%.

Problem 2

Two way to solve the problem.

1. We consider the sample space

$$\Omega = \big\{\{H,H\},\{H,T\},\{T,T\}\big\}$$

with probability measure

$$\mathbb{P}\{H,H\} = \mathbb{P}\{T,T\} = \frac{1}{4} \quad \text{and} \quad \mathbb{P}\{H,T\} = \frac{1}{2}.$$

Then the event B : « *One coin is Head* » is given by

$$B = \big\{\{H,H\},\{H,T\}\big\}.$$

So

$$\mathbb{P}(\{H,H\} \mid B) = \frac{\mathbb{P}(\{H,H\} \cap B)}{\mathbb{P}(B)} = \frac{\mathbb{P}\{H,H\}}{\mathbb{P}\{H,T\} + \mathbb{P}\{H,H\}} = \frac{1/4}{3/4} = \frac{1}{3},$$

(and not $\frac{1}{2}$ as all of you wrote). Notice that in this case, the event « *At least one coin is Head* » is in fact B

2. We'll consider

$$\Omega = \{(H,H),(H,T),(T,H),(T,T)\},$$

with probability measure

$$\mathbb{P}\{\omega\} = \frac{1}{|\Omega|} = \frac{1}{4}.$$

So now the event B : « *One coin is head* » is given by

$$B = \big\{(T,H),(H,T),(H,H)\big\},$$

and thus

$$\mathbb{P}(\{H,H\} \mid B) = \frac{\mathbb{P}\{H,H\}}{\mathbb{P}(B)}.$$

In this case,

$$\mathbb{P}(B) = \frac{|B|}{|\Omega|} = \frac{3}{4}.$$

So at the end, we get

$$\mathbb{P}(\{H,H\} \mid B) = \frac{1/4}{3/4} = \frac{1}{3},$$

as well.

Problem 3

The sample space is given by

$$\Omega = \{(i,j,k) \mid i,j,k \in \{1,\dots,6\}\},$$

with probability measure

$$\mathbb{P}\{\omega\} = \frac{1}{|\Omega|} = \frac{1}{6^3}.$$

Let consider the events A :« *All dices are differents* » and B :« *One dice is 6* ». By definition of conditional expectation

$$\mathbb{P}(B \mid A) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}.$$

We have that

$$A \cap B = \{(6,j,k) \mid j \neq k \neq 6 \neq j\} \cup \{(j,6,k) \mid j \neq k \neq 6 \neq j\} \cup \{(j,k,6) \mid j \neq k \neq 6 \neq j\}$$

$$A = \{(i,j,k) \mid i \neq j \neq k \neq i\}.$$

So

$$|A \cap B| = 3 \cdot 1 \cdot 5 \cdot 4 \quad \text{and} \quad |A| = 6 \cdot 5 \cdot 4.$$

Therefore

$$\mathbb{P}(B \mid A) = \frac{|A \cap B|/|\Omega|}{|B|/|\Omega|} = \frac{|A \cap B|}{|B|} = \frac{1}{2}.$$

Problem 6

Let A and B independents. Then

$$\mathbb{P}(A \cap B^c) = \mathbb{P}(A) - \mathbb{P}(A \cap B) \underset{(*)}{=} \mathbb{P}(A) - \mathbb{P}(A)\mathbb{P}(B) = \mathbb{P}(A)(1 - \mathbb{P}(B)) = \mathbb{P}(A)\mathbb{P}(B^c),$$

where $(*)$ come from the independence of A and B. Therefore A and B^c are independent. The proof of the independence of A^c and B^c goes the same and is left to the reader.

Problem 8

Let A_i the events « *He has an accident the i^{th} day* » where $i = 1, \cdot, 365$. The A_i's are independents. Then so are the A_i^c (by problem 6).

$$\mathbb{P}(\text{« \textit{At least one accident this year} »}) = 1 - \mathbb{P}(\text{« \textit{No accident this year} »})$$

$$= 1 - \mathbb{P}\left(\bigcap_{i=1}^{365} A_i^c\right) \underset{(*)}{=} 1 - \mathbb{P}(A_1^c)^{365} = 1 - \left(\frac{499}{500}\right)^{365},$$

where $(*)$ come from the independence of the A_i's.