

Title:

Overcoming the curse of dimensionality: from nonlinear Monte Carlo to deep artificial neural networks

Abstract:

Partial differential equations (PDEs) are among the most universal tools used in modelling problems in nature and man-made complex systems. For example, stochastic PDEs are a fundamental ingredient in models for nonlinear filtering problems in chemical engineering and weather forecasting, deterministic Schroedinger PDEs describe the wave function in a quantum physical system, deterministic Hamiltonian-Jacobi-Bellman PDEs are employed in operations research to describe optimal control problems where companies aim to minimise their costs, and deterministic Black-Scholes-type PDEs are highly employed in portfolio optimization models as well as in state-of-the-art pricing and hedging models for financial derivatives. The PDEs appearing in such models are often high-dimensional as the number of dimensions, roughly speaking, corresponds to the number of all involved interacting substances, particles, resources, agents, or assets in the model. For instance, in the case of the above mentioned financial engineering models the dimensionality of the PDE often corresponds to the number of financial assets in the involved hedging portfolio. Such PDEs can typically not be solved explicitly and it is one of the most challenging tasks in applied mathematics to develop approximation algorithms which are able to approximatively compute solutions of high-dimensional PDEs. Nearly all approximation algorithms for PDEs in the literature suffer from the so-called "curse of dimensionality" in the sense that the number of required computational operations of the approximation algorithm to achieve a given approximation accuracy grows exponentially in the dimension of the considered PDE. With such algorithms it is impossible to approximatively compute solutions of high-dimensional PDEs even when the fastest currently available computers are used. In the case of linear parabolic PDEs and approximations at a fixed space-time point, the curse of dimensionality can be overcome by means of Monte Carlo approximation algorithms and the Feynman-Kac formula. In this talk we introduce new nonlinear Monte Carlo algorithms for high-dimensional nonlinear PDEs. We prove that such algorithms do indeed overcome the curse of dimensionality in the case of a general class of semilinear parabolic PDEs and we thereby prove, for the first time, that a general semilinear parabolic PDE with a nonlinearity depending on the PDE solution can be solved approximatively without the curse of dimensionality.