Martingale problems and their connection to Markov processes

Martingale problems appear naturally in the study of stochastic processes. A famous example is Lévy's characterization of Brownian motion which allows us to characterize Brownian motion by just two martingales. In this lecture series, we are, more generally, interested in the connection between Markov processes and martingales. It turns out that there is a natural way to associate a family of martingales with a Markov process. We will see that this family of martingales encodes a lot of information about the original Markov process. Moreover, we will also study the converse question, that is, given a family of martingales can we use it to construct a Markov process? As an application of the presented material, we will derive an existence and uniqueness result for solutions to stochastic differential equations.