Seventh Bielefeld-SNU Joint Workshop in Mathematics
Abstracts

February 26 – March 1, 2019

Tuesday, February 26

Metastability of interacting particle systems

Claudio Landim

We present the metastable behavior of certain interacting particle systems. We first examine the evolution of the condensate in a class of zero-range processes evolving in a finite set, and describe its nucleation phase. In the second part of the talk, we consider the metastable behavior of the Ising model under the Kawasaki dynamics.

Spectral analysis of non-local Markov generators

Yuri Kondratiev

We study generators of random walks in the continuum and their potential perturbations. Main problems which will be discussed: the existence and properties of ground states, resolvent bounds, heat kernels for non-local generators. Applications to certain real world models will be a subject of our considerations.

Emergence of anomalous flocking in the fractional Cucker-Smale model

Jinwook Jung

In this talk, we study the emergent behaviors of the Cucker-Smale (C-S) ensemble under the interplay of memory effect and flocking dynamics. As a mathematical model incorporating aforementioned interplay, we introduce the fractional C-S model which can be obtained by replacing the usual time derivative by the Caputo fractional time derivative. For the proposed fractional C-S model, we provide a sufficient framework which admits the emergence of anomalous flocking at the algebraic rate and an l^2-stability estimate with respect to initial data. We also provide several numerical examples and compare them with our theoretical results. Furthermore, we discuss several intrinsic randomness in the model and present its stochastic variants and some possible estimates.

Nonlinear perturbations of evolution systems in scales of Banach spaces

Oleksandr Kutovyi

A variant of the abstract Cauchy-Kovalevskaya theorem is considered. We prove existence and uniqueness of classical solutions to the nonlinear, non-autonomous initial value problem

$$\frac{\mathrm{d}u(t)}{\mathrm{d}t} = A(t)u(t) + B(u(t), t), \quad u(0) = x$$

in a scale of Banach spaces. Here $A(t)$ is the generator of an evolution system acting in a scale of Banach spaces and $B(u,t)$ obeys an Ovcyannikov-type bound. Continuous dependence of the solution with respect to $A(t)$, $B(u,t)$ and x is proved. The results are applied to the Kimura-Maruyama equation for the mutation-selection balance model. This yields a new insight in the construction and uniqueness question for nonlinear Fokker-Planck equations related with interacting particle systems in the continuum.

Scaling limit of metastable diffusion processes

Insuk Seo

In this presentation, we explain a new methodology for the analysis of metastable processes. For continuous processes exhibiting metastable behavior, the martingale approach developed by Beltran and Landim is hard to be applied. Our new methodology is based on the analysis of Poisson equations related to the generator of the corresponding metastable process. This presentation is based on the joint works with Claudio Landim, and with Fraydoun Rezakhanlou.

Homogenization of zero order convolution type operators with random coefficients

Andrey Piatnitski

The talk will focus on homogenization problem for a symmetric convolution type operator with a random ergodic statistically homogeneous coefficient. Assuming that the kernel of this operator is integrable and satisfies natural moment conditions we show that under the diffusive scaling the family of rescaled operators admits almost sure homogenization, and that it converges to a second order elliptic operator with deterministic constant coefficients. This is a joint work with Elena Zhizhina.

Noise-induced synchronization in circulant networks of weakly coupled oscillators

Barbara Gentz

We will address the question of synchronization in finite-size systems of coupled harmonic oscillators. For commensurate oscillators and circulant coupling structures we will show that weak multiplicative-noise coupling can amplify some of the system's eigenmodes and thus lead to asymptotic eigenmode synchronization.
Reference: PhD thesis of Christian Wiesel, University of Bielefeld, 2018

Wednesday, February 27

Heat kernel estimates for time fractional equations

Panki Kim

In this talk, we discuss general time fractional equations and give their probabilistic representations of solutions. We derive sharp two-sided estimates for fundamental solutions of a large family of time fractional equations in metric measure spaces.

Estimates and stability of heat kernels for symmetric jump processes with general mixed polynomial growths on metric measure space

Jaehun Lee

In this talk, we establish the stability of two-sided heat kernel estimates for symmetric jump Markov processes on metric measure spaces that satisfies general volume doubling condition. Our results cover Markov processes whose jumping density has mixed polynomial growths. In particular, our scaling function may not be comparable to the function which gives the growth of jumps. To obtain sharp two-sided heat kernel estimates, we need additional condition on the metric measure space, which is called the chain condition. If underlying metric measure space allows a conservative diffusion process which has the transition density with certain type of sub-Gaussian estimates, our scaling function depends on not only jump density but also walking dimension of metric measure space.

On the division problem for wave maps

Sebastian Herr

Tataru's approach to wave maps in the critical Besov space is revisited. A new solution to the division problem via bilinear Fourier restriction estimates and atomic function spaces will be presented.

New regularity results for dispersive PDE on tori via shorttime Strichartz estimates

Robert Schippa

In Euclidean space linear dispersive PDE are characterized by a decay estimate and conservation of mass. On compact manifolds mass is still conserved, but the decay estimate can not hold globally in time because this would contradict mass conservation. In now already classical works by Burq-Gerard-Tzvetkov (2004) and Staffilani-Tataru (2002) was pointed out how localization in time to small frequency dependent time intervals recovers the dispersive estimate. These considerations are combined with shorttime function spaces utilized in Ionescu-Kenig-Tataru (2008) for solutions on Euclidean space. New regularity results for dispersive PDE on tori from the preprints arXiv:1704.07174 and arXiv:1810.04406 are presented.

Thursday, February 28

CLT for the maximum of branching random walk in random environment and random F-KPP equation

Jiří Černý

The behaviour of the maximal particle of branching random walk have been subject to intensive research recently. It is natural to ask how these properties change when a spatially dependent random branching rates are introduced to the process. In my presentation, I will describe the first results in this direction, in particular a CLT for the position of the maximal particle, and explain their consequences for other models of interest: the randomized Fisher-KPP equation, and the parabolic Anderson model.

Estimates of Dirichlet heat kernels for unimodal Lévy processes with low intensity of small jumps

Jaehoon Kang

In this talk, we discuss transition density functions for pure jump unimodal Lévy processes killed upon leaving an open set D. In analytic point of view, the transition density function is Dirichlet heat kernel for infinitesimal generator of the corresponding Lévy processes. Under some mild assumptions on the Lévy density, we establish two-sided Dirichlet heat kernel estimates when the open set D is $C^{1,1}$. This result covers the case that the Lévy densities of unimodal Lévy processes are regularly varying functions whose indices are equal to the Euclidean dimension. This is a joint work with Soobin Cho and Panki Kim.

Boundary regularity for nonlocal operators with kernels of variable orders

Minhyun Kim

We study the boundary regularity of solutions of the Dirichlet problem for the nonlocal operator with a kernel of variable orders. Since the order of differentiability of the kernel is not represented by a single number, we consider the generalized Hölder space. We prove that there exists a unique viscosity solution of $Lu = f$ in D, $u = 0$ in $\mathbb{R}^n \setminus D$, where D is a bounded $C^{1,1}$ open set, and that the solution u satisfies $u \in C^V(D)$ and $u/V(d_D) \in C^\alpha(D)$ with the uniform estimates, where V is the renewal function and $d_D(x) = \mathrm{dist}(x, \partial D)$.

Mosco convergence of nonlocal to local quadratic forms

Guy Fabrice Foghem Gounoue

We study sequences of nonlocal quadratic forms and function spaces that are related to Markov jump processes in bounded domains with a Lipschitz boundary. Our aim is to show the convergence of these forms to local quadratic forms of gradient type. Under suitable conditions we establish the convergence in the sense of Mosco. Our framework allows to study bounded and unbounded nonlocal operators at the same time. Moreover, we prove that smooth functions with compact support are dense in the nonlocal function spaces under consideration.

Semilinear PDE on fractals

Michael Hinz

We first give a short introduction to analysis on fractal state spaces and the typical difficulties involved. We then discuss gradient operators and certain semilinear PDEs and briefly explain two recent results. The first is a metric graph approximation for a Burgers type equation, the second is a law of large numbers for an exclusion process on a fractal.

Sobolev spaces and calculus of variations on fractals

Melissa Meinert

In this talk, we will review $p-$energies and $(1, p)-$Sobolev spaces for fractals and metric measure spaces that carry a local Dirichlet form. These Sobolev spaces are then used to generalize some basic results from the calculus of variations, such as the existence of minimizers of convex functionals.

Backward SDEs with quadratic growth

Khaled Bahlali

We introduce a domination argument which asserts that: if we can dominate the parameters of a quadratic backward stochastic differential equation (QBSDE) with continuous generator from above and from below by those of two BSDEs having ordered solutions, then also the original QBSDE has a solution. This abstract result allows us to solve quadratic BSDEs whose coefficient H is continuous and satisfies $|H(t,y,z)| \leq \alpha_t + \beta_t|y| + \gamma_t|z| + f(|y|)|z|^2$, where α_t, β_t, γ_t are positive processes and f a real valued locally bounded function. We get the existence of solutions in the following cases:

1) when f is globally integrable, locally bounded and the terminal value ξ belongs to $\mathbb{L}^2$ (*Bahlali-Eddahbi-Ouknine 2013 and 2017*).

2) when f is continuous and increasing and the terminal value ξ satisfies some integrability condition (*Bahlali 2017/2018*).

3) when $f(y) = \frac{1}{y}$ and ξ belongs to $\mathbb{L}^3$ and is strictly positive or strictly negative (*Bahlali-Tangpi 2018*).

These kind of BSDEs are related to semilinear PDEs whose nonlinearity is dominated by $a + b|u| + c|\nabla_x u| + f(|u|)|\nabla_x u|^2$. As a consequence, we deduce the existence of viscosity solutions these type of PDEs.

Homogenization of biased convolution type operators

Elena Zhizhina

I will tell about results from our recent work with A. Piatnitski, where we studied homogenization of the parabolic problem for integral convolution type operators with a non-symmetric jump kernel in a periodic elliptic medium. It was shown that the homogenization result holds in moving coordinates. We found the corresponding effective velocity and proved that the limit operator is a second order parabolic operator with constant coefficients. We also considered the behaviour of the effective velocity in the case of small antisymmetric perturbations of a symmetric kernel, in particular we showed that the Einstein relation holds for the studied periodic environment.

Friday, March 1

Local laws for Hermitian and products of non-Hermitian random matrices

Friedrich Götze

We shall present recent results about local approximations of the spectrum of Hermitian and products of non-Hermitian random random matrices. In the Hermitian case we describe related results for the delocalization of eigenvectors and rigidity of eigenvalues. We shall describe some of the technical tools, which allow to prove these results under nearly four moments only. This is joint work with A. Naumov and A. Tikhomirov.

Distribution of algebraic numbers and their connection to the roots of random polynomials

Anna Gusakova

The question of the distribution of real and complex algebraic numbers has been considered during the last few years and a tight relation between the distribution of algebraic numbers and the distribution of the zeros of random polynomials has been found. The typical problem is to find the asymptotic formula for the number of algebraic numbers lying in some domain D with given degree n and height bounded by Q when Q tends to infinity. In this talk we obtain such formula for D being the arc of the unit circle in the complex plane using results on the distribution of zeros of random trigonometric polynomial.

Based on a joint work with Friedrich Götze, Dmitry Zaporozhets and Zakhar Kabluchko.

Existence, uniqueness and ergodic properties for time-homogeneous Itô-SDEs with locally integrable drifts and Sobolev diffusion coefficients

Gerald Trutnau

Using elliptic and parabolic regularity results in L^p-spaces and generalized Dirichlet form theory, we construct for every starting point weak solutions to SDEs in $\mathbb{R}^d$ up to their explosion times including the following conditions. For arbitrary but fixed $p > d$ the diffusion coefficient $A = (a_{ij})$ is locally uniformly strictly elliptic with functions $a_{ij} \in H^{1,p}_{loc}(\mathbb{R}^d)$ and the drift coefficient $\mathbf{G} = (g_1, \ldots, g_d)$ consists of functions $g_i \in L^p_{loc}(\mathbb{R}^d)$. The solution is by construction a Hunt process with continuous sample paths on the one-point compactification of $\mathbb{R}^d$ and by a known local well-posedness result pathwise unique and strong up to its explosion time. Just under the given assumptions we show irreducibility and the $L^{[1,\infty]}(\mathbb{R}^d, m)$-strong Feller property of its transition function, and the $L^{[q,\infty]}(\mathbb{R}^d, m)$-strong Feller property, $q = \frac{dp}{d+p} \in (d/2, p/2)$, of its resolvent, which both include the classical strong Feller property. We present moment inequalities and classical-like non-explosion criteria for the solution which lead to pathwise uniqueness results up to infinity under presumably optimal general non-explosion conditions. We further present explicit conditions for recurrence and ergodicity, including existence as well as uniqueness of invariant measures. This is joint work with Haesung Lee (Seoul National University).

Existence and regularity of invariant measures, transition functions and time homogeneous Itô-SDEs

Haesung Lee

We show existence of an invariant measure m for a large class of elliptic second order partial differential operators with local Sobolev diffusion coefficient and locally square integrable drift. Subsequently, we derive regularity properties of the corresponding semigroup which is defined in $L^p(\mathbb{R}^d, m)$, $p \in [1, \infty]$, including the classical strong Feller property and classical irreducibility. This leads to a transition function of a Hunt process that is explicitly identified as a solution to an SDE. Properties of this Hunt process, like non-explosion, moment inequalities, recurrence and transience, as well as ergodicity, including the uniqueness of m, can then be studied using the derived analytical tools. In contrast to previous results, where regularity theory of equations whose solutions are measures is used, we use regularity theory for divergence forms, which allows the coefficients to be more singular. This is joint work with Gerald Trutnau (Seoul National University).

Parabolic stochastic partial differential equations driven by Lévy noise

Andre Schenke

We study second-order quasi-linear SPDEs on C^1 domains subjected to Lévy noise. We prove uniqueness and existence of solutions in (weighted) Sobolev spaces and obtain L_p and Hölder estimates of both the solution and its gradient. This is a joint work with K.H. Kim.

On Cherny's results in infinite dimensions: A theorem dual to Yamada-Watanabe

Marco Rehmeier

We prove that joint uniqueness in law and the existence of a strong solution imply pathwise uniqueness for variational solutions to stochastic partial differential equations of the form

$$\mathrm{d}X_t = b(t,X)\mathrm{d}t + \sigma(t,X)\mathrm{d}W_t, \;\; t \geq 0,$$

and show that for such equations uniqueness in law is equivalent to joint uniqueness in law. Here W is a cylindrical Wiener process in a separable Hilbert space U and the equation is considered in a Gelfand triple $V \subseteq H \subseteq E$, where H is some separable (infinite-dimensional) Hilbert space. This generalizes the corresponding results of A. Cherny for the case of finite-dimensional equations.

Title: t.b.a.

Michael Röckner

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