

Seventh Bielefeld-SNU Joint Workshop in Mathematics

Abstracts

February 26 – March 1, 2019

Mosco convergence of nonlocal to local quadratic forms

Guy Fabrice Foghem Gounoue

We study sequences of nonlocal quadratic forms and function spaces that are related to Markov jump processes in bounded domains with a Lipschitz boundary. Our aim is to show the convergence of these forms to local quadratic forms of gradient type. Under suitable conditions we establish the convergence in the sense of Mosco. Our framework allows to study bounded and unbounded nonlocal operators at the same time. Moreover, we prove that smooth functions with compact support are dense in the nonlocal function spaces under consideration.

On the division problem for wave maps

Sebastian Herr

Tataru's approach to wave maps in the critical Besov space is revisited. A new solution to the division problem via bilinear Fourier restriction estimates and atomic function spaces will be presented.

Emergence of anomalous flocking in the fractional Cucker-Smale model

Jinwook Jung

In this talk, we study the emergent behaviors of the Cucker-Smale (C-S) ensemble under the interplay of memory effect and flocking dynamics. As a mathematical model incorporating aforementioned interplay, we introduce the fractional C-S model which can be obtained by replacing the usual time derivative by the Caputo fractional time derivative. For the proposed fractional C-S model, we provide a sufficient framework which admits the emergence of anomalous flocking at the algebraic rate and an l^2 -stability estimate with respect to initial data. We also provide several numerical examples and compare them with our theoretical results. Furthermore, we discuss several intrinsic randomness in the model and present its stochastic variants and some possible estimates.

Estimates of Dirichlet heat kernels for unimodal Lévy processes with low intensity of small jumps

Jaehoon Kang

In this talk, we discuss transition density functions for pure jump unimodal Lévy processes killed upon leaving an open set D . In analytic point of view, the transition density function is Dirichlet heat kernel for infinitesimal generator of the corresponding Lévy processes. Under some mild assumptions on the Lévy density, we establish two-sided Dirichlet heat kernel estimates when the open set D is $C^{1,1}$. This result covers the case that the Lévy densities of unimodal Lévy processes are regularly varying functions whose indices are equal to the Euclidean dimension. This is a joint work with Soobin Cho and Panki Kim.

Boundary regularity for nonlocal operators with kernels of variable orders

Minhyun Kim

We study the boundary regularity of solutions of the Dirichlet problem for the nonlocal operator with a kernel of variable orders. Since the order of differentiability of the kernel is not represented by a single number, we consider the generalized Hölder space. We prove that there exists a unique viscosity solution of $Lu = f$ in D , $u = 0$ in $\mathbb{R}^n \setminus D$, where D is a bounded $C^{1,1}$ open set, and that the solution u satisfies $u \in C^V(D)$ and $u/V(d_D) \in C^\alpha(D)$ with the uniform estimates, where V is the renewal function and $d_D(x) = \text{dist}(x, \partial D)$.

Heat kernel estimates for time fractional equations

Panki Kim

In this talk, we discuss general time fractional equations and give their probabilistic representations of solutions. We derive sharp two-sided estimates for fundamental solutions of a large family of time fractional equations in metric measure spaces.

Spectral analysis of non-local Markov generators

Yuri Kondratiev

We study generators of random walks in the continuum and their potential perturbations. Main problems which will be discussed: the existence and properties of ground states, resolvent bounds, heat kernels for non-local generators. Applications to certain real world models will be a subject of our considerations.

Metastability of interacting particle systems

Claudio Landim

We present the metastable behavior of certain interacting particle systems. We first examine the evolution of the condensate in a class of zero-range processes evolving in a finite set, and describe its nucleation phase. In the second part of the talk, we consider the metastable behavior of the Ising model under the Kawasaki dynamics.

Existence and regularity of invariant measures, transition functions and time homogeneous Itô-SDEs

Haesung Lee

We show existence of an invariant measure m for a large class of elliptic second order partial differential operators with local Sobolev diffusion coefficient and locally square integrable drift. Subsequently, we derive regularity properties of the corresponding semigroup which is defined in $L^p(\mathbb{R}^d, m)$, $p \in [1, \infty]$, including the classical strong Feller property and classical irreducibility. This leads to a transition function of a Hunt process that is explicitly identified as a solution to an SDE. Properties of this Hunt process, like non-explosion, moment inequalities, recurrence and transience, as well as ergodicity, including the uniqueness of m , can then be studied using the derived analytical tools. In contrast to previous results, where regularity theory of equations whose solutions are measures is used, we use regularity theory for divergence forms, which allows the coefficients to be more singular. This is joint work with Gerald Trutnau (Seoul National University).

Sobolev spaces and calculus of variations on fractals

Melissa Meinert

In this talk, we will review p -energies and $(1, p)$ -Sobolev spaces for fractals and metric measure spaces that carry a local Dirichlet form. These Sobolev spaces are then used to generalize some basic results from the calculus of variations, such as the existence of minimizers of convex functionals.

Homogenization of zero order convolution type operators with random coefficients

Andrey Piatnitski

The talk will focus on homogenization problem for a symmetric convolution type operator with a random ergodic statistically homogeneous coefficient. Assuming that the kernel of this operator is integrable and satisfies natural moment conditions we show that under the diffusive scaling the family of rescaled operators admits almost sure homogenization, and that it converges to a second order elliptic operator with deterministic constant coefficients. This is a joint work with Elena Zhizhina.

On Cherny's results in infinite dimensions: A theorem dual to Yamada-Watanabe

Marco Rehmeier

We prove that joint uniqueness in law and the existence of a strong solution imply pathwise uniqueness for variational solutions to stochastic partial differential equations of the form

$$dX_t = b(t, X)dt + \sigma(t, X)dW_t, \quad t \geq 0,$$

and show that for such equations uniqueness in law is equivalent to joint uniqueness in law. Here W is a cylindrical Wiener process in a separable Hilbert space U and the equation is considered in a Gelfand triple $V \subseteq H \subseteq E$, where H is some separable (infinite-dimensional) Hilbert space. This generalizes the corresponding results of A. Cherny for the case of finite-dimensional equations.

Parabolic stochastic partial differential equations driven by Lévy noise

Andre Schenke

We study second-order quasi-linear SPDEs on C^1 domains subjected to Lévy noise. We prove uniqueness and existence of solutions in (weighted) Sobolev spaces and obtain L_p and Hölder estimates of both the solution and its gradient. This is a joint work with K.H. Kim.

New regularity results for dispersive PDE on tori via shorttime Strichartz estimates

Robert Schippa

In Euclidean space linear dispersive PDE are characterized by a decay estimate and conservation of mass. On compact manifolds mass is still conserved, but the decay estimate can not hold globally in time because this would contradict mass conservation. In now already classical works by Burq-Gerard-Tzvetkov (2004) and Staffilani-Tataru (2002) was pointed out how localization in time to small frequency dependent time intervals recovers the dispersive estimate. These considerations are combined with shorttime function spaces utilized in Ionescu-Kenig-Tataru (2008) for solutions on Euclidean space. New regularity results for dispersive PDE on tori from the preprints arXiv:1704.07174 and arXiv:1810.04406 are presented.

Homogenization of biased convolution type operators

Elena Zhizhina

I will tell about results from our recent work with A. Piatnitski, where we studied homogenization of the parabolic problem for integral convolution type operators with a non-symmetric jump kernel in a periodic elliptic medium. It was shown that the homogenization result holds in moving coordinates. We found the corresponding effective velocity and proved that the limit operator is a second order parabolic operator with constant coefficients. We also considered the behaviour of the effective velocity in the case of small antisymmetric perturbations of a symmetric kernel, in particular we showed that the Einstein relation holds for the studied periodic environment.